

See-saw and Grand Unification

Goran Senjanović

International Centre for Theoretical Physics, 34100 Trieste, Italy

I review the profound connection between the see-saw mechanism for neutrino masses and grand unification. This connection points naturally towards $SO(10)$ grand unified theory. The emphasis here is on the supersymmetric theory, but I also discuss salient features of its split supersymmetry version and ordinary non-supersymmetric $SO(10)$. Particular attention is paid to the crucial issue of the minimal such theory, i.e. the question of the Higgs sector needed to break $SO(10)$ down to the Minimal Supersymmetric Standard Model or the Standard Model. Some essential features of the see-saw mechanism are clarified, in particular its precise origin at the high scale.

Prelude

I have been asked by the organizers of the SEESAW25 to review the see-saw mechanism in connection with grand unification. Due to the enormous body of work in the field, neither time nor space allow me to do a complete job. Instead I focus here on the work regarding pure grand unification, the work connected with the search for the minimal grand unified theory, both supersymmetric and not.

I am forced thus to omit some important issues such as doublet triplet splitting, grand unification in extra dimension, non minimal models, fermion mass textures, and more. For some complementary reviews of these topics (and not only) see: [1, 2, 3, 4].

I. THE PARAMETERS

We know today [5] that neutrinos are massive and we know the two mass differences that correspond to the atmospheric and solar neutrino oscillations

$$\Delta m_A^2 \simeq (2.5 \pm 0.6) 10^{-3} \text{eV}^2; \quad \Delta m_\odot^2 \simeq (8.2 \pm 0.6) 10^{-5} \text{eV}^2 \quad (1)$$

The corresponding mixing angles are

$$\theta_A = 45^\circ \pm 6^\circ; \quad \theta_\odot = 32.5^\circ \pm 2.5^\circ \quad (2)$$

From (1), the mass of the heaviest neutrino has a lower limit

$$m_\nu^{\max} \geq 5 \times 10^{-2} \text{eV} \quad (3)$$

We also know from β decay that

$$m_{\nu_e} \leq 2.2 \text{eV} \quad (95\% c.l.) \quad (4)$$

and from cosmological data we know that the sum of neutrino masses is small

$$\sum m_\nu \lesssim 0.4 \text{eV} - 1.7 \text{eV} \quad (5)$$

Thus, even if degenerate, neutrinos are very light, $m_\nu \lesssim (0.15 \text{eV} - 0.6 \text{eV})$. How to understand so small neutrino masses? A simple answer lies in the see-saw mechanism.

II. SEE-SAW IN THE STANDARD MODEL

The see-saw mechanism [6] of neutrino masses in the context of the Standard Model (SM), is obtained by adding a right-handed neutrino and writing the most general $SU(2)_L \times U(1)_Y$ Yukawa Lagrangean

$$L_y(\nu) = y_D \bar{\nu}_L \phi \nu_R + \frac{1}{2} M_R \nu_R^T C^{-1} \nu_R + h.c. \quad (6)$$

In our symbolic notation, y_D is a Yukawa matrix in flavor space. From (1), at least two light neutrinos are massive, and thus one needs at least two right-handed neutrinos. Having in mind grand unification with quark-lepton symmetry a

la Pati-Salam as a natural framework to study neutrino masses, in what follows I assume three right-handed neutrinos and suppress generation indices.

Now, (6) creates a mess, unless $M_R \gg y_D \langle \phi \rangle$. This is quite natural though, since M_R is a gauge invariant quantity and thus expected to be very large: $M_R \gg M_W$. The principle that gauge invariant quantities lie much above the scale of the breaking of the symmetry in question lies at the heart of the see-saw mechanism. In what follows, I will stick to it consistently.

Since $y_D \leq 1$, with $M_R \gg \langle \phi \rangle$ one gets automatically small neutrino masses

$$M_\nu^I = m_D^T M_R^{-1} m_D \quad (7)$$

where $m_D \equiv y_D \langle \phi \rangle$ and I stands for the type I see-saw, which has become the common name for this realization of small neutrino masses.

Alternatively, you could add to the SM a $SU(2)_L$ triplet Δ , with $B - L(\Delta) = 2$, and the Yukawa couplings

$$L_y(\nu) = y_\nu \ell_L^T C \Delta \ell_L + h.c. \quad (8)$$

where ℓ_L stands for the leptonic doublets. The triplet gets a non-vanishing vev $\langle \Delta \rangle \simeq \frac{M_W^2}{M_\Delta}$ if the triplet mass $M_\Delta \gg M_W$. The same principle as before ensures small neutrino masses [7]

$$M_\nu^{II} \simeq y_\nu \frac{M_W^2}{M_\Delta} \quad (9)$$

where the superscript II stands for the type II see-saw as is commonly called.

In the SM we cannot distinguish the two mechanisms for $M_R, M_\Delta \gg M_W$. They are both simply the expression of the effective operator analysis which tells us that the leading $SU(2)_L \times U(1)_Y$ Yukawa coupling is of dimension 5 [8]

$$L_y^{\text{eff}}(\nu) = f \frac{1}{M} (\ell^T \sigma_2 \Phi) C (\phi^T \sigma_2 \ell) \quad (10)$$

where $M \gg M_W$ and f is a matrix in generation space. Hence small, see-saw like suppressed neutrino masses

$$M_\nu \simeq f \frac{M_W^2}{M} \quad (11)$$

Obviously, both type I and type II see-saw are of the form (11), as they have to be. There is no sense in trying to distinguish type I from type II in the SM, not without any new physics being invoked. After all, if $y_D \propto y_\nu \propto M_R$, we will even have the same flavor structure for neutrino mass matrices. In order to study this issue, we must go beyond the SM.

Our task is highly nontrivial. In order that the see-saw mechanism be tested, in order that it be a theory, we need first of all to know the origin of the mechanism: is it type I, type II or something else? [55] Since M_R has to be very large (unless Yukawa couplings are extremely small), it is natural to consider grand unification as a framework of new large mass scales and of quark-lepton unification which sheds light on y_D (and/or y_ν).

III. HOW TO INCORPORATE SEE-SAW IN GUTS ?

This question is intimately tied up with the fundamental issue of the choice of *the* grand unified theory. After three decades of grand unification, there is no consensus today of what the theory is.

First, the symmetry reasoning. In the SM, $B - L$ is an accidental, anomaly free $U(1)$ global symmetry. It is of course broken by M_R (or M in the effective operator language). If you believe in an accidental global $B - L$, then the $SU(5)$ direction is the natural one, since this phenomenon persists.

On the other hand, the fact that $B - L$ is anomaly free makes a strong case for its gauging. This would in turn induce $(B - L)^3$ anomaly; the simplest remedy is to introduce the right-handed neutrinos (one per generation). The natural setting is then provided by Left-Right ($L - R$) symmetric theories [10] where right-handed neutrinos are a must and $B - L$ has a simple physical interpretation from the electric charge formula

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2} \quad (12)$$

The new scale M_R is then the scale of $SU(2)_R$, or better to say $L - R$ (parity) symmetry breaking. It is important to stress that both type I and type II see-saw emerge naturally in this case and are deeply connected. This route points towards Pati-Salam quark-lepton unification and $SO(10)$ as a grand unified theory (GUT).

In the next two sections, I go through both $SU(5)$ and $SO(10)$ supersymmetric theories. Of course, $SU(5)$ needs low energy supersymmetry (or split supersymmetry[11]), whereas the same cannot be said of the $SO(10)$ theory. I will thus discuss both supersymmetric and ordinary $SO(10)$.

IV. GRANDUNIFICATION: SU(5)

As remarked before, SU(5) is a natural theory if you give up gauging $B - L$. The minimal SU(5) theory fails, for the gauge couplings do not unify. With low energy supersymmetry the couplings unify as predicted more than two decades ago[12].

The minimal theory with only 5_H (and $\bar{5}_H$ in SUSY), predicts $m_d = m_e$ at M_{GUT} , generation by generation [13]. Whereas $m_b = m_\tau$ works well, for other generations $|m_\mu| \simeq 3|m_s|$, $|m_e| \simeq 1/3|m_d|$ is needed (again, at M_{GUT}). This is easily achieved without any change in the theory, by adding higher dimensional operators suppressed by $1/M_{Pl}$. The theory loses then its predictivity in determining precisely M_{GUT} and τ_p , but is saved from being ruled out by a too fast $d = 5$ proton decay[14]. Alternatively, one could add more Higgs superfields, say 45_H as in the Georgi-Jarlskog[15] approach.

What about neutrino masses in $SU(5)$? You could choose from three simple possibilities, none of them very appealing:

1. Add right-handed neutrinos, SU(5) singlets. In this case M_R is a gauge invariant quantity, and by the principle of naturalness $M_R \gg M_{GUT}$. This is no good, since then $m_\nu \ll M_W^2/M_{GUT} \simeq 10^{-3}\text{eV}$, which is too small to explain the solar and especially the atmospheric neutrino data.
2. Add a $SU(2)_L$ triplet as before, with Δ contained in 15_H , a two index symmetric Higgs superfield. With the same principle as above, you reach the same conclusion.
3. You could write a higher-dimensional operator *a la* Weinberg [8]

$$O_5 = f \frac{1}{M_\nu} \bar{5}_F \bar{5}_F 5_H 5_H \quad (13)$$

with $M_\nu \gg M_{GUT}$, say $M_\nu \sim M_{Pl}$ [16]. Strictly speaking, (1) and (2) correspond to this, as in the SM case discussed before. Again, the Planck scale suppression is too large to account for the atmospheric or solar neutrino data. Such terms can be relevant though for small splittings in the case of degenerate neutrinos[17].

All this does not prove that we must give up on SU(5). After all, we can fine-tune M_R , the trouble is that the Yukawas are arbitrary, as much as in the SM. Such a theory, with all the tree-level and $1/M_{Pl}$ corrections to Yukawa couplings needed to correct fermionic mass relations, has too many parameters. In order to pursue this direction, one should go beyond SU(5) and invoke extra family horizontal symmetries, discrete, global or local (for a review and references see[2]). Instead, we turn to SO(10) which is tailor fit for a theory of fermion masses.

V. SO(10): THE MINIMAL THEORY OF MATTER AND GAUGE COUPLING UNIFICATION

There are a number of features that make SO(10) special:

1. a family of fermions is unified in a 16-dimensional spinorial representation; this in turn predicts the existence of right-handed neutrinos
2. $L - R$ symmetry is a finite gauge transformation in the form of charge conjugation. This is a consequence of both left-handed fermions f_L and its charged conjugated counterparts $(f^c)_L \equiv C \bar{f}_R^T$ residing in the same representation 16_F .
3. in the supersymmetric version, matter parity $M = (-1)^{3(B-L)}$, equivalent to the R-parity $R = M(-1)^{2S}$, is a gauge transformation [18], a part of the center Z_4 of SO(10). It simply reads $16 \rightarrow -16$, $10 \rightarrow 10$. Its fate depends then on the pattern of symmetry breaking (or the choice of Higgs fields); it turns out that in the renormalizable version of the theory R-parity remains exact at all energies[19, 20]. The lightest supersymmetric partner (LSP) is then stable and is a natural candidate for the dark matter of the universe.
4. its other maximal subgroup, besides $SU(5) \times U(1)$, is $SO(4) \times SO(6) = SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry of Pati and Salam. It explains immediately the somewhat mysterious relations $m_d = m_e$ (or $m_d = 1/3 m_e$) of SU(5).
5. the unification of gauge couplings can be achieved with or without supersymmetry.
6. the minimal renormalizable version (with no higher dimensional $1/M_{Pl}$ terms) offers a simple and deep connection between $b - \tau$ unification and a large atmospheric mixing angle in the context of the type II see-saw [21].

In order to understand some of these results, and in order to address the issue of construction of the theory, we turn now to the Yukawa sector.

A. Yukawa sector

Fermions belong to the spinor representation 16_F [22]. From

$$16 \times 16 = 10 + 120 + \overline{126} \quad (14)$$

the most general Yukawa sector in general contains 10_H , 120_H and $\overline{126}_H$, respectively the fundamental vector representation, the three-index antisymmetric representation and the five-index antisymmetric and anti-self-dual representation. $\overline{126}_H$ is necessarily complex, supersymmetric or not; 10_H and $\overline{126}_H$ Yukawa matrices are symmetric in generation space, while the 120_H one is antisymmetric.

Understanding fermion masses is easier in the Pati-Salam language of one of the two maximal subgroups of $SO(10)$, $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$ (the other being $SU(5) \times U(1)$). Let us decompose the relevant representations under G_{PS}

$$\mathbf{16} = (4, 2, 1) + (\bar{4}, 1, 2) \quad (15)$$

$$\mathbf{10} = (1, 2, 2) + (6, 1, 1) \quad (16)$$

$$\mathbf{120} = (1, 2, 2) + (6, 3, 1) + (6, 1, 3) + (15, 2, 2) + (10, 1, 1) + (\overline{10}, 1, 1) \quad (17)$$

$$\overline{\mathbf{126}} = (\overline{10}, 3, 1) + (10, 1, 3) + (15, 2, 2) + (6, 1, 1) \quad (18)$$

Clearly, the see-saw mechanism, whether type I or II, requires $\overline{126}$: it contains both $(10, 1, 3)$ whose vev gives a mass to ν_R (type I), and $(\overline{10}, 3, 1)$, which contains a color singlet, $B - L = 2$ field Δ_L , that can give directly a small mass to ν_L (type II). A reader familiar with the $SU(5)$ language sees this immediately from the decomposition under this group

$$\overline{\mathbf{126}} = 1 + 5 + 15 + \overline{45} + 50 \quad (19)$$

The 1 of $SU(5)$ belongs to the $(10, 1, 3)$ of G_{PS} and gives a mass for ν_R , while 15 corresponds to the $(\overline{10}, 3, 1)$ and gives the direct mass to ν_L .

Of course, $\overline{126}_H$ can be a fundamental field, or a composite of two $\overline{16}_H$ fields, or can even be induced as a two-loop effective representation built out of a 10_H and two gauge 45-dim representations. In what follows I shall discuss carefully all three possibilities.

Normally the light Higgs is chosen to be the smallest one, 10_H . Since $\langle 10_H \rangle = \langle (1, 2, 2) \rangle_{PS}$ is a $SU(4)_c$ singlet, $m_d = m_e$ follows immediately, independently of the number of 10_H you wish to have. Thus we must add either 120_H or $\overline{126}_H$ or both in order to correct the bad mass relations. Both of these fields contain $(15, 2, 2)_{PS}$, and its vev gives the relation $m_e = -3m_d$.

As $\overline{126}_H$ is needed anyway for the see-saw, it is natural to take this first. The crucial point here is that in general $(1, 2, 2)$ and $(15, 2, 2)$ mix through $\langle (10, 1, 3) \rangle$ [23] and thus the light Higgs is a mixture of the two. In other words, $\langle (15, 2, 2) \rangle$ in $\overline{126}_H$ is in general non-vanishing [56]. It is rather appealing that 10_H and $\overline{126}_H$ may be sufficient for all the fermion masses, with only two sets of symmetric Yukawa coupling matrices.

B. An instructive failure

Before proceeding, let me emphasize the crucial point of the necessity of 120_H or $\overline{126}_H$ in the charged fermion sector on an instructive failure: a simple and beautiful model by Witten [24]. The model is non-supersymmetric and the SUSY lovers may place the blame for the failure here. It uses $\langle 16_H \rangle$ in order to break $B - L$, and the “light” Higgs is 10_H . Witten noticed an ingenious and simple way of generating an effective mass for the right-handed neutrino, through a two-loop effect which gives

$$M_{\nu_R} \simeq y_{up} \left(\frac{\alpha}{\pi} \right)^2 M_{GUT} \quad (20)$$

where one takes all the large mass scales, together with $\langle 16_H \rangle$, of the order M_{GUT} . Since $\langle 10_H \rangle = \langle (1, 2, 2)_{PS} \rangle$ preserves quark-lepton symmetry, it is easy to see that

$$\begin{aligned} M_\nu &\propto M_u \\ M_e &= M_d \\ M_u &\propto M_d \end{aligned} \tag{21}$$

so that $V_{\text{lepton}} = V_{\text{quark}} = 1$. The model fails badly. Is it yet another example of beautiful theories killed by the ugly facts of nature?

The original motivation of Witten was a desire to know the scale of M_{ν_R} and increase M_ν , at that time neutrino masses were expected to be larger. But the real achievement of this simple, minimal SO(10) theory is the predictivity of the structure of M_{ν_R} and thus M_ν . It is an example of a good, albeit wrong theory: it fails because it predicts.

What is the moral behind the failure? The main problem, in my opinion, was to ignore the fact that with only 10_H already charged fermion masses fail. As I have argued repeatedly, one needs a $\overline{126}_H$ (or a 120_H), one way or another, a lesson we keep in what follows.

There is a nice way to save Witten's mechanism though. In order to work it leads automatically to GUT scale heavy sfermions and light, order TeV, gauginos and Higgsinos. I postpone its discussion to the section devoted to this, so called split supersymmetry, scenario.

VI. SUPERSYMMETRIC SO(10) GUT

In supersymmetry 10_H is necessarily complex and the bidoublet $(1, 2, 2)$ in 10_H contains the two Higgs doublets of the MSSM, with the vevs v^u and v^d in general different: $\tan \beta \equiv v^u/v^d \neq 1$ in general. In order to study the physics of SO(10), we need to know what the theory is, i.e. its Higgs content. There are two orthogonal approaches to the issue, as we discuss now.

A. Small representations

The idea: take the smallest Higgs fields (least number of fields, not of representations) that can break SO(10) down to the MSSM and give realistic fermion masses and mixings. The following fields are both necessary and sufficient

$$45_H, 16_H + \overline{16}_H, 10_H \tag{22}$$

It all looks simple and easy to deal with, but the superpotential becomes extremely complicated. First, at the renormalizable level it is too simple. The pure Higgs and the Yukawa superpotential at the renormalizable level take the form

$$\begin{aligned} W_H = & m_{45} 45_H^2 + m_{16} 16_H \overline{16}_H + \lambda_1 16_H \Gamma^2 \overline{16}_H 45_H \\ & m_{10} 10_H^2 + \lambda_2 16_H \Gamma 16_F 10_H + \lambda_3 \overline{16}_H \Gamma \overline{16}_H 10_H \end{aligned} \tag{23}$$

$$W_y = y_{10} 16_F \Gamma 16_F 10_H \tag{24}$$

where Γ stands for the Clifford algebra matrices of SO(10), $\Gamma_1 \dots \Gamma_{10}$, and the products of Γ 's are written in a symbolic notation (both internal and Lorentz charge conjugation are omitted).

Clearly, both W_H and W_y are insufficient. The fermion mass matrices would be completely unrealistic and the vevs $\langle 45_H \rangle, \langle 6_H \rangle, \langle \overline{16}_H \rangle$ would all point in the SU(5) direction. Thus, one adds non-renormalizable operators

$$\begin{aligned} \Delta W_H = & \frac{1}{M_{Pl}} [(45_H^2)^2 + 45_H^4 + (16_H \overline{16}_H)^2 + (16_H \Gamma^2 \overline{16}_H)^2 + (16_H \Gamma^4 \overline{16}_H)^2 \\ & + (16_H \Gamma 16_H)^2 + (16_H \Gamma^5 16_H)^2 + \{16_H \rightarrow \overline{16}_H\} \\ & + 16_H \Gamma^4 \overline{16}_H 45_H^2 + 16_H \Gamma^3 \overline{16}_H 45_H 10_H + \{16_H \rightarrow \overline{16}_H\}] \end{aligned} \tag{25}$$

$$\begin{aligned} \Delta W_y = & \frac{1}{M_{Pl}} [16_F \Gamma 16_F 16_H \Gamma 16_H + \{16_H \rightarrow \overline{16}_H\} \\ & 16_F \Gamma^3 16_F 45_H 10_H + 16_F \Gamma^5 16_F \overline{16}_H \Gamma^5 \overline{16}_H] \end{aligned} \tag{26}$$

where I take for simplicity all the couplings to be unity; there are simply too many of them. The large number of Yukawa couplings means very little predictivity.

The way out is to *add flavor symmetries* and to play the texture game and thus reduce the number of couplings. This in a sense goes beyond grand unification and appeals to new physics at M_{Pl} and/or new symmetries (see e.g. [25]).

To me, maybe the least appealing aspect of this approach is the loss of R (matter) parity due to 16_H and $\overline{16}_H$; it must be postulated by hand as much as in the MSSM.

On the positive side, it is an asymptotically free theory and one can work in the perturbative regime all the way up to M_{Pl} . While this sounds nice, I am not sure what it means in practice. It would be crucial if you were able to make high precision determination of M_{GUT} or m_T , the mass of colored triplets responsible for $d = 5$ proton decay. The trouble is that the lack of knowledge of the superpotential couplings is sufficient even in the minimal $SU(5)$ theory to prevent this task; in $SO(10)$ it gets even worse.

Maybe more relevant is the fact that in this scenario $M_R \simeq M_{GUT}^2/M_{Pl} \simeq 10^{13} - 10^{14} \text{ GeV}$, which fits nicely with the neutrino masses via see-saw. Furthermore, see-saw can be considered “clean”, of the pure type I, since the type II effect is suppressed by $1/M_{Pl}$. Most important, the $m_b \simeq m_\tau$ relation from (24) is maintained due to small $1/M_{Pl}$ effects relevant only for the first two generations.

Now, the higher dimensional operators can be mimicked by the inclusion of singlets when they are integrating out (assuming them much heavier than M_{GUT} , as expected by the gauge principle). This paves the way for model building if one is willing to fine-tune their masses to lie below M_{GUT} , and this way one can get the double type I see-saw formula (see e.g. [26]). Recently Barr [27] has shown how in a particular case one can obtain the see-saw formula linear in y_D (and not quadratic as usual). Although obtained in a completely different manner, this is what happens in the Witten’s model, and thus in my opinion does not really represent a new type of see-saw. Of course, this allows for different models of neutrino masses and mixings. In order to stick to minimal theories, I refrain here from discussing this and similar proposals; this does not imply that they are without merit.

B. Big is Better approach

The non-renormalizable operators in reality mean invoking new physics beyond grand unification. This may be necessary, but still, one should be more ambitious and try to use the renormalizable theory only. This means large representations necessarily: at least $\overline{126}_H$ is needed in order to give the mass to ν_R (in supersymmetry, one must add 126_H). The consequence is the loss of asymptotic freedom above M_{GUT} , the coupling constants grow large at the scale $\Lambda_F \simeq 10M_{GUT}$ [28]. To me this is a priori neither good nor bad, but if it bothers you, you should skip the rest of the section.

Once we accept large representations, we should minimize their number. The minimal theory contains, on top of 10_H , 126_H and $\overline{126}_H$, also 210_H [29, 30, 31, 32] with the decomposition

$$210_H = (1, 1, 1)_- + (15, 1, 1)_+ + (15, 1, 3) + (15, 3, 1) + (6, 2, 2) + (10, 2, 2) + (\overline{10}, 2, 2) \quad (27)$$

where the $-(+)$ subscript denotes the properties of the color singlets under charge conjugation.

The Higgs superpotential is remarkably simple

$$W_H = m_{210}(210_H)^2 + m_{126}\overline{126}_H 126_H + m_{10}(10_H)^2 + \lambda(210_H)^3 \\ + \eta 126_H \overline{126}_H 210_H + \alpha 10_H 126_H 210_H + \overline{\alpha} 10_H \overline{126}_H 210_H \quad (28)$$

and the Yukawa one even simpler

$$W_Y = y_{10} 16_F \Gamma 16_F 10_H + y_{126} 16_F \Gamma^5 16_F \overline{126}_H \quad (29)$$

Remarkably enough, this may be sufficient, without any higher dimensional operators; however, the situation is not completely clear.

There is a small number of parameters: $3 + 6 \times 2 = 15$ real Yukawa couplings, and 11 real parameters in the Higgs sector. In this sense the theory can be considered as the minimal supersymmetric GUT in general [32]. As usual, I am not counting the parameters associated with the SUSY breaking terms.

The nicest feature of this program (and the best justification for the use of large representations) is the following. Besides the $\langle(10, 1, 3)\rangle$ which gives masses to the ν_R ’s, also the $\langle(15, 2, 2)\rangle$ in $\overline{126}_H$ gets a vev [23, 30]. Approximately

$$\langle(15, 2, 2)\rangle_{\overline{126}} \simeq \frac{M_{PS}}{M_{GUT}} \langle(1, 2, 2)\rangle \quad (30)$$

with $M_{PS} = \langle(15, 2, 2)\rangle$ being the scale of $SU(4)_c$ symmetry breaking. In SUSY, $M_{PS} \leq M_{GUT}$ and thus one can have correct mass relations for the charged fermions.

What is lost, though, is the $b - \tau$ unification, i.e. with $\langle(15, 2, 2)\rangle_{\overline{126}} \neq 0$, $m_b = m_\tau$ at M_{GUT} becomes an accident. However, in the case of type II see-saw, there is a profound connection between $b - \tau$ unification and a large atmospheric mixing angle. The fermionic mass matrices are obtained from (29)

$$\begin{aligned} M_u &= v_{10}y_{10} + v_{126}^u y_{126} , \\ M_d &= v_{10}y_{10} + v_{126}^d y_{126} , \\ M_e &= v_{10}y_{10} - 3v_{126}^d y_{126} , \\ M_{\nu_D} &= v_{10}y_{10} - 3v_{126}^u y_{126} , \end{aligned} \tag{31}$$

$$M_{\nu_R} = y_{126}\langle(10, 1, 3)\rangle , \tag{32}$$

$$M_{\nu_L} = y_{126}\langle(\overline{10}, 3, 1)\rangle , \tag{33}$$

$$\tag{34}$$

where $\langle(\overline{10}, 3, 1)\rangle \simeq M_W^2/M_{GUT}$ provides a direct (type II) see-saw mass for light neutrinos. The form in (31) is readily understandable, if you notice that $\langle(1, 2, 2)\rangle$ is a $SU(4)_c$ singlet with $m_q = m_\ell$, and $\langle(15, 2, 2)\rangle$ is a $SU(4)_c$ adjoint, with $m_\ell = -3m_q$. The vevs of the bidoublets are denoted by v^u and v^d as usual.

Now, suppose that type II dominates, or $M_\nu \propto y_{126} \propto M_e - M_d$, so that

$$M_\nu \propto M_e - M_d \tag{35}$$

Let us now look at the 2nd and 3rd generations first. In the basis of diagonal M_e , and for the small mixing ϵ_{de}

$$M_\nu \propto \begin{pmatrix} m_\mu - m_s & \epsilon_{de} \\ \epsilon_{de} & m_\tau - m_b \end{pmatrix} \tag{36}$$

obviously, large atmospheric mixing can only be obtained for $m_b \simeq m_\tau$ [21].

Of course, there was no reason whatsoever to assume type II see-saw. Actually, we should reverse the argument: the experimental fact of $m_b \simeq m_\tau$ at M_{GUT} , and large θ_{atm} seem to favor the type type II see-saw. It can be shown, in the same approximation of 2-3 generations, that type I cannot dominate: it gives a small θ_{atm} [33]. This gives hope to disentangle the nature of the see-saw in this theory. As a check, it can be shown that the two types of see-saw are really inequivalent [33].

The three generation numerical studies supported a type II see-saw [57] with the interesting prediction of a large θ_{13} and a hierarchical neutrino mass spectrum [36]. Somewhat better fits are obtained with a small contribution of 120_H [37] or higher dimensional operators [38].

I wish to stress an important feature of this programme. Since 126 ($\overline{126}$) is invariant under matter parity, R parity remains exact at all energies and thus the lightest supersymmetric particle is stable and a natural candidate for the dark matter.

1. Mass scales

In $SO(10)$ we have in principle more than one scale above M_W (and Λ_{SUSY}): the GUT scale, the Pati-Salam scale where $SU(4)_c$ is broken, the L-R scale where parity (charge conjugation) is broken, the scales of the breaking of $SU(2)_R$ and $U(1)_{B-L}$. Of course, these may be one and the same scale, as expected with low-energy supersymmetry. This solution is certainly there, since the gauge couplings of the MSSM unify successfully and encourage the single step breaking of $SO(10)$.

Is there any room for intermediate mass scales in SUSY $SO(10)$? It is certainly appealing to have an intermediate see-saw mass scale M_R , between $10^{12} - 10^{15}$ GeV or so. In the non-renormalizable case, with 16_H and $\overline{16}_H$, this is precisely what happens: $M_R \simeq cM_{GUT}^2/M_{Pl} \simeq c(10^{13} - 10^{14})$ GeV. In the renormalizable case, with 126_H and $\overline{126}_H$, one needs to perform a renormalization group study using unification constraints. While this is in principle possible, in practice it is hard due to the large number of fields. The stage has recently been set, for all the particle masses were computed [39, 40], and the preliminary studies show that the situation may be under control [41]. It is interesting that the existence of intermediate mass scales lowers the GUT scale [39, 42] (as was found before in models with 54_H and 45_H [20]), allowing for a possibly observable $d = 6$ proton decay.

Notice that a complete study is basically impossible. In order to perform the running, you need to know particle masses precisely. Now, suppose you stick to the principle of minimal fine-tuning. As an example, you fine-tune the mass of the W and Z in the SM, then you know that the Higgs mass and the fermion masses are at the same scale

$$m_H = \frac{\sqrt{\lambda}}{g} m_W , \quad m_f = \frac{y_f}{g} m_W \tag{37}$$

where λ is a ϕ^4 coupling, and y_f an appropriate fermionic Yukawa coupling. Of course, you know the fermion masses in the SM model, and you know $m_H \simeq m_W$.

In an analogous manner, at some large scale m_G a group G is broken and there are usually a number of states that lie at m_G , with masses

$$m_i = \alpha_i m_G \quad (38)$$

where α_i is an approximate dimensionless coupling. Most renormalization group studies typically argue that $\alpha_i \simeq O(1)$ is natural, and rely on that heavily. In the SM, you could then take $m_H \simeq m_W$, $m_f \simeq m_W$; while reasonable for the Higgs, it is nonsense for the fermions (except for the top quark).

In supersymmetry *all* the couplings are of Yukawa type, i.e. self-renormalizable, and thus taking $\alpha_i \simeq O(1)$ may be as wrong as taking all $y_f \simeq O(1)$. While a possibly reasonable approach when trying to get a qualitative idea of a theory, it is clearly unacceptable when a high-precision study of M_{GUT} is called for.

2. Proton decay

As you know, $d = 6$ proton decay gives $\tau_p(d = 6) \propto M_{GUT}^4$, while $(d = 5)$ gives $\tau_p(d = 5) \propto M_{GUT}^2$. In view of the discussion above, the high-precision determination of τ_p appears almost impossible in $SO(10)$ (and even in $SU(5)$). Preliminary studies [43] indicate fast $d = 5$ decay as expected.

You may wonder if our renormalizable theory makes sense at all. After all, we are ignoring the higher dimensional operators of order $M_{GUT}/M_{Pl} \simeq 10^{-2} - 10^{-3}$. If they are present with the coefficients of order one, we can forget almost everything we said about the predictions, especially in the Yukawa sector. However, we actually know that the presence of $1/M_{Pl}$ operators is not automatic (at least not with the coefficients of order 1). Operators of the type (in symbolic notation)

$$O_5^p = \frac{c}{M_{Pl}} 16_F^4 \quad (39)$$

are allowed by $SO(10)$ and they give

$$O_5^p = \frac{c}{M_{Pl}} [(QQQL) + (Q^c Q^c Q^c L^c)] \quad (40)$$

These are the well-known $d = 5$ proton decay operators, and for $c \simeq O(1)$ they give $\tau_p \simeq 10^{23} yr$. Agreement with experiment requires

$$c \leq 10^{-6} \quad (41)$$

Could this be a signal that $1/M_{Pl}$ operators are small in general? Alternatively, you need to understand why just this one is to be so small. It is appealing to assume that this may be generic; if so, neglecting $1/M_{Pl}$ contributions in the study of fermion masses and mixings is fully justified.

3. Leptogenesis

The see-saw mechanism provides a natural framework for baryogenesis through leptogenesis, obtained by the out-of-equilibrium decay of heavy right-handed neutrinos[44]. This works nicely for large M_R , in a sense too nicely. Already type I see-saw works by itself, but the presence of the type II term makes things more complicated [45]. One cannot be a priori sure whether the decay of right-handed neutrinos or the heavy Higgs triplets is responsible for the asymmetry, although the hierarchy of Yukawa couplings points towards ν_R decay. In the type II see-saw, the most natural scenario is the ν_R decay, but with the triplets running in the loops [46]. This and related issues are now under investigation[47].

VII. SUPERSYMMETRY: IS IT REALLY NEEDED?

In the last two decades, and especially after its success with gauge coupling unification, grand unification by an large got tied up with low energy supersymmetry. This is certainly well motivated, since supersymmetry is the only mechanism in field theory which controls the gauge hierarchy. On the other hand, I hope to have convinced

you that the right grand unified theory should be based on $SO(10)$, not $SU(5)$. If so, gauge coupling unification needs no supersymmetry whatsoever. It only says that there must be intermediate scales[48], such as Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$ or Left-Right $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, between M_W and M_{GUT} (the $SU(5)$ route is ruled out). An oasis or two in the desert is always welcome.

Thus if we accept the fine-tuning, as we seem to be forced in the case of the cosmological constant, we can as well study the ordinary, non-supersymmetric version of the theory. In this context the idea of the cosmic attractors[49] as the solution to the gauge hierarchy becomes extremely appealing. It needs no supersymmetry whatsoever, and enhances the motivation for ordinary grand unified theories. In what follows I discuss some essential features of a possible minimal such theory.

Let me stick to a purely renormalizable theory for the sake of simplicity and predictivity. The minimal such theory is based on 210_H , $\overline{126}_H$ (no need for 126_H as in SUSY) and a “light” 10_H . In this case, the theory is asymptotically free, and thus there is no advantage with small representations. A purely renormalizable theory can alternatively be built with 45_H and 54_H instead of 210_H . Notice that 45_H would not suffice: it turns out to preserve $SU(5)$ [50], and $\overline{126}_H$ must preserve it in order not to break the SM symmetry.

Intermediate mass scales help lower the masses of ν_R , but create potential problems for the charged fermions on the other hand. We have seen in the supersymmetric version that the light Higgs is a mixture of $(1, 2, 2)$ in 10_H and $(15, 2, 2)$ in $\overline{126}_H$. This is crucial if one is to get correct mass relations between down quarks and charged leptons. These fields can mix through $(15, 1, 1)$ in 45_H or 210_H , and $(10, 1, 3)$ in $\overline{126}_H$, either by the trilinear couplings $(1, 2, 2), (15, 2, 2), (15, 1, 1)$ or the quartic ones $(1, 2, 2), (15, 2, 2), (15, 1, 1)^2$; $(1, 2, 2), (15, 2, 2), (10, 1, 3)^2$. In other words

$$\langle(15, 2, 2)\rangle \simeq \left(\frac{M_I}{M_{GUT}}\right)^n \langle(1, 2, 2)\rangle \quad (42)$$

where $n = 1, 2$ for trilinear and quartic mixings respectively (which depends on what the GUT scale fields are). Since $\langle(15, 2, 2)\rangle$ is needed for the second generation, with $m_\mu/M_W \simeq 10^{-3}$, we have the constraint

$$\left(\frac{M_I}{M_{GUT}}\right)^n \geq 10^{-3} \quad (43)$$

This can be used to eliminate the single intermediate scale chain of symmetry breaking[51].

Another important difference with the SUSY situation lies in the Yukawa sector where now, in the minimal theory, 10_H is real. This implies $v_{10}^u = v_{10}^d$ and fitting fermionic masses and mixings becomes impossible[51].

If it does fail, one could just add another 10_H ; this means unfortunately another y_{10} . You can avoid it by postulating a Peccei-Quinn symmetry

$$16_F \rightarrow e^{i\alpha} 16_F, \quad 10_H \rightarrow e^{-2i\alpha} 10_H, \quad \overline{126}_H \rightarrow e^{-2i\alpha} \overline{126}_H \quad (44)$$

with 10_H now complex. This can give you naturally the axionic dark matter, at the expense of introducing additional $\overline{126}$ (or 126) in order to break both $B - L$ and $U(1)_{PQ}$. Although somewhat unappealing and against the rules of sticking to the pure $SO(10)$, the loss of neutralinos as the dark matter may necessitate this. If you dislike this fact, simply work with two 10_H ’s and two y_{10} ’s. Adding another 10_H is especially mild in the type II see-saw since the relation $M_d - M_e \propto M_\nu$ is independent of the number of 10_H ’s. Thus $b - \tau$ unification is still connected with a large θ_{atm} as in the supersymmetric case. In recent years not much attention was devoted to the ordinary $SO(10)$, except for the work of the Napoli group (see e.g. Ref.[52]).

VIII. SPLIT SUPERSYMMETRY

If the need for the perturbative control of the weak scale is given up, there appears an interesting alternative of split supersymmetry as I mentioned before. This scenario which imagines a large supersymmetry breaking scale, with heavy sfermions and light, order TeV, gauginos and Higgsinos has recently attracted a lot of attention [53]. Actually in the minimal $SU(5)$ this is the only alternative to the low energy supersymmetry; the only trouble is that $SU(5)$ is not a good theory of massive neutrinos. In $SO(10)$ we have also an option of no supersymmetry as we have just seen. How to know what the TeV effective physics, relevant for LHC, is? Is it possible that a GUT answers the question? We recently came up with a theory that does just that[54]. It is based on a Witten’s radiative seesaw scenario and its generalization to charged fermions. All one needs is to have two 10_H fields, or one 10_H and one 120_H in the Yukawa sector. The main prediction of the theory is the split supersymmetry with scalar masses of the order of the GUT scale (except of course for a light Higgs). To see that one needs to extend the radiative mechanism to a strongly broken supersymmetric theory, with righthanded neutrino masses

$$M_{\nu_R} \approx \left(\frac{\alpha}{\pi}\right)^2 Y_{10} \frac{M_R^2}{M_{GUT}} f\left(\frac{\tilde{m}}{M_{GUT}}\right), \quad (45)$$

where \tilde{m} is the scale of supersymmetry breaking, or in other words the difference between the scalar and fermion masses of the same supermultiplet. This is valid only for \tilde{m} not above M_{GUT} . The function $f(x) \rightarrow 0$ when $x \rightarrow 0$ and $f(x) = O(1)$ if $x = O(1)$.

Due to the two loops suppression the only way to have large enough righthanded neutrino masses is through single step symmetry breaking $M_R \approx M_{GUT}$ and the large $\tilde{m} \approx M_{GUT}$. The unification constraints with no intermediate scale require then light gauginos and Higgsinos. Thus, independently of the details of the realistic Yukawa sector, one is forced to the split supersymmetry picture.

IX. SUMMARY AND OUTLOOK

The see-saw mechanism has emerged in recent years as the simplest and the most natural way of explaining small neutrino masses. It simply means adding right-handed neutrinos to the Standard Model, allowing them to have large gauge invariant masses M_R which break $B - L$ symmetry and through Dirac Yukawa couplings y_D with left-handed neutrinos give the latter non-vanishing, but small masses. As appealing as this may be, as useless it is in practice, The lack of knowledge of M_R and y_D leaves neutrino masses arbitrary.

I have argued in this talk that the most natural framework for see-saw is grand unification, a theory of large mass scale and the stage for the $q - \ell$ symmetry which can hopefully connect y_D and perhaps M_R with quark Yukawa couplings.

If you accept this, then by now you should be convinced that the right GUT is based on $SO(10)$ gauge group. Basically all you ever wanted is there: right-handed neutrinos, Pati-Salam quark-lepton symmetry, charge conjugation as a L-R symmetry and much more. The trouble is that physics depends not only on the gauge symmetry, but almost as much on the choice of the Higgs sector. It is here that the practitioners cannot agree yet and most attention is devoted to two rather orthogonal approaches. One insists on perturbativity all the way to the Planck scale and chooses small representations: 16_H (+ $\overline{16}_H$ in SUSY) and 45_H . This program then uses $1/M_{Pl}$ operators to generate the physically acceptable superpotential; uses textures to simplify the theory and thus appeal to physics beyond grand unification.

The other approach sticks to the pure $SO(10)$ theory with no $1/M_{Pl}$ operators. This means large representation $\overline{126}_H$ (+ 126_H in SUSY), 210_H ; strong couplings in SUSY at $\lambda_F = 10M_{GUT}$, but is blessed with a small number of couplings and is a complete theory of matter and non-gravitational interactions. This program is good enough to be testable. Especially appealing is the version with the type 2 seesaw since it offers a deep connection between a large atmospheric mixing angle and b-tau unification; it furthermore predicts a large 1-3 leptonic mixing angle.

Strictly speaking in $SO(10)$ one needs no supersymmetry at all, at least not for the reasons of unification. If the unnaturalness of the small Higgs mass is accepted, the nonsupersymmetric version with large representation maintains all the good features discussed above, and remains asymptotically free.

Another, maybe even more interesting possibility, is a strongly broken supersymmetry. Namely, if one sticks to the minimal theory with a 16_H Higgs, then a radiative seesaw mechanism works very nicely as long as supersymmetry is split; one has a clear prediction of light gauginos and Higgsinos, but superheavy, order GUT scale, sfermions. This is a rare example when an inner structure of a high-energy theory sheds light on the TeV physics relevant for LHC without any assumptions about the naturalness.

I guess the main message of this short review is a caution when discussing the see-saw mechanism. By itself, it is only an aesthetically appealing scenario devoid of practical use. It makes sense to discuss it only in the context of a well-defined theory based on firm physical principles.

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-
- [1] R. N. Mohapatra, arXiv:hep-ph/0211252.
 - [2] G. Altarelli and F. Feruglio, arXiv:hep-ph/0405048.
 - [3] C. H. Albright, Int. J. Mod. Phys. A **18** (2003) 3947.
 - [4] J. C. Pati, arXiv:hep-ph/0407220.
 - [5] For some recent reviews see e.g. B. Bajc, F. Nesti, G. Senjanovic and F. Vissani, “Perspectives in neutrino physics,” *Proceedings of 17th Rencontres de Physique de la Vallée d’Aoste*, La Thuile, 9-15 Mar 2003, M. Greco ed., page 103-143; S. M. Bilenky, C. Giunti, J. A. Grifols and E. Masso, Phys. Rept. **379** (2003) 69; V. Barger, D. Marfatia and K. Whisnant, Int. J. Mod. Phys. E **12** (2003) 569. A. Y. Smirnov, Int. J. Mod. Phys. A **19** (2004) 1180.
 - [6] P. Minkowski, Phys. Lett. B **67** (1977) 421. T. Yanagida, proceedings of the *Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, 1979, eds. A. Sawada, A. Sugamoto, KEK Report No. 79-18, Tsukuba. S. Glashow, in *Quarks and Leptons, Cargèse 1979*, eds. M. Lévy. et al., (Plenum, 1980, New York). M. Gell-Mann, P. Ramond, R. Slansky, proceedings of the *Supergravity Stony Brook Workshop*, New York, 1979, eds. P. Van Nieuwenhuizen, D. Freeman (North-Holland, Amsterdam). R. Mohapatra, G. Senjanović, Phys.Rev.Lett. **44** (1980) 912
 - [7] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181** (1981) 287. R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23** (1981) 165.
 - [8] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.
 - [9] T. Hambye, Y. Lin, A. Notari, M. Papucci and A. Strumia, Nucl. Phys. B **695** (2004) 169.
 - [10] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275. R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 2558. G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12** (1975) 1502. G. Senjanović, Nucl. Phys. B **153** (1979) 334.
 - [11] N. Arkani-Hamed and S. Dimopoulos, arXiv:hep-th/0405159.
 - [12] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Rev. D **24** (1981) 1681. L.E. Ibáñez, G.G. Ross, Phys. Lett. B **105** (1981) 439. M.B. Einhorn, D.R. Jones, Nucl. Phys. B **196** (1982) 475. W. Marciano, G. Senjanović, Phys.Rev.D **25** (1982) 3092.
 - [13] M. S. Chanowitz, J. R. Ellis and M. K. Gaillard, Nucl. Phys. B **128**, 506 (1977). A. J. Buras, J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **135** (1978) 66.
 - [14] B. Bajc, P. Pérez and G. Senjanović, Phys. Rev. D **66** (2002) 075005. B. Bajc, P. Fileviez Pérez and G. Senjanović, arXiv:hep-ph/0210374. D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B **661** (2003) 62.
 - [15] H. Georgi and C. Jarlskog, Phys. Lett. B **86**, 297 (1979).
 - [16] R. Barbieri, J. R. Ellis and M. K. Gaillard, Phys. Lett. B **90** (1980) 249. E. K. Akhmedov, Z. G. Berezhiani and G. Senjanović, Phys. Rev. Lett. **69** (1992) 3013.
 - [17] E. K. Akhmedov, Z. G. Berezhiani, G. Senjanovic and Z. j. Tao, Phys. Rev. D **47**, 3245 (1993) [arXiv:hep-ph/9208230].
 - [18] R. N. Mohapatra, *Phys. Rev. D* **34**, 3457 (1986). A. Font, L. E. Ibáñez and F. Quevedo, *Phys. Lett. B* **228**, 79 (1989). S. P. Martin, *Phys. Rev. D* **46**, 2769 (1992).
 - [19] C.S. Aulakh, K. Benakli, G. Senjanović, Phys. Rev. Lett. **79** (1997) 2188. C. S. Aulakh, A. Melfo and G. Senjanović, Phys. Rev. D **57**, 4174 (1998). C. S. Aulakh, A. Melfo, A. Rašin and G. Senjanović, Phys. Lett. B **459** (1999) 557.
 - [20] C. S. Aulakh, B. Bajc, A. Melfo, A. Rašin and G. Senjanović, Nucl. Phys. B **597** (2001) 89.
 - [21] B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. Lett. **90** (2003) 051802.
 - [22] For useful reviews on spinors in SO(2N) see R. N. Mohapatra and B. Sakita, Phys. Rev. D **21** (1980) 1062 and F. Wilczek and A. Zee, Phys. Rev. D **25** (1982) 553. See also P. Nath and R. M. Syed, Nucl. Phys. B **618** (2001) 138 and C. S. Aulakh and A. Girdhar, arXiv:hep-ph/0204097.
 - [23] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).
 - [24] E. Witten, Phys. Lett. B **91** (1980) 81.
 - [25] K.S. Babu, S.M. Barr, Phys. Rev. D **51** (1995) 2463. K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. **74** (1995) 2418. G.R. Dvali, S. Pokorski, Phys. Lett. B **379** (1996) 126. S.M. Barr, S. Raby, Phys. Rev. Lett. **79** (1997) 4748. Z. Chacko, R.N. Mohapatra, Phys. Rev. D **59** (1999) 011702. K. S. Babu, J. C. Pati and F. Wilczek, Nucl. Phys. B **566** (2000) 33. C. H. Albright and S. M. Barr, Phys. Rev. Lett. **85** (2000) 244. M. C. Chen and K. T. Mahanthappa, Phys. Rev. D **62** (2000) 113007. Z. Berezhiani and A. Rossi, Nucl. Phys. B **594** (2001) 113. R. Kitano and Y. Mimura, Phys. Rev. D **63** (2001) 016008.
 - [26] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986).
 - [27] S. M. Barr, Phys. Rev. Lett. **92** (2004) 101601.
 - [28] C. S. Aulakh, arXiv:hep-ph/0210337.
 - [29] C.S. Aulakh, R.N. Mohapatra, Phys. Rev. D **28** (1983) 217.
 - [30] T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B **115** (1982) 26.
 - [31] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. D **30** (1984) 1052. X. G. He and S. Meljanac, Phys. Rev. D **41** (1990) 1620. D. G. Lee, Phys. Rev. D **49** (1994) 1417. D. G. Lee and R. N. Mohapatra, Phys. Rev. D **51** (1995) 1353.
 - [32] C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. B **588**, 196 (2004).
 - [33] B. Bajc, G. Senjanović and F. Vissani, arXiv:hep-ph/0402140.
 - [34] K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. D **64** (2001) 053015. T. Fukuyama and N. Okada, JHEP **0211** (2002)

011.

- [35] L. Lavoura, Phys. Rev. D **48** (1993) 5440. B. Brahmachari and R. N. Mohapatra, Phys. Rev. D **58** (1998) 015001. K. y. Oda, E. Takasugi, M. Tanaka and M. Yoshimura, Phys. Rev. D **59** (1999) 055001.
- [36] H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B **570** (2003) 215. H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Rev. D **68** (2003) 115008.
- [37] S. Bertolini, M. Frigerio and M. Malinsky, arXiv:hep-ph/0406117. W. M. Yang and Z. G. Wang, arXiv:hep-ph/0406221. B. Dutta, Y. Mimura and R. N. Mohapatra, arXiv:hep-ph/0406262.
- [38] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. D **69** (2004) 115014.
- [39] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Rev. D **70** (2004) 035007.
- [40] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, arXiv:hep-ph/0401213. C. S. Aulakh and A. Girdhar, arXiv:hep-ph/0204097.
- [41] C. S. Aulakh and A. Girdhar, arXiv:hep-ph/0405074.
- [42] H. S. Goh, R. N. Mohapatra and S. Nasri, arXiv:hep-ph/0408139.
- [43] H. S. Goh, R. N. Mohapatra, S. Nasri and S. P. Ng, Phys. Lett. B **587** (2004) 105. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, JHEP **0409** (2004) 052.
- [44] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
- [45] See the talk by T. Hambye at this conference, and references therein.
- [46] T. Hambye and G. Senjanović, Phys. Lett. B **582** (2004) 73. S. Antusch and S. F. King, Phys. Lett. B **597** (2004) 199. P. J. O'Donnell and U. Sarkar, Phys. Rev. D **49** (1994) 2118. E. Ma and U. Sarkar, Phys. Rev. Lett. **80** (1998) 5716. G. Lazarides and Q. Shafi, Phys. Rev. D **58** (1998) 071702.
- [47] P. Fileviez-Pérez, T. Hambye and G. Senjanović, work in preparation.
- [48] See e.g. D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak and M. K. Parida, Phys. Rev. D **31** (1985) 1718.
- [49] G. Dvali and A. Vilenkin, arXiv:hep-th/0304043. G. Dvali, arXiv:hep-th/0410286.
- [50] L. F. Li, Phys. Rev. D **9** (1974) 1723.
- [51] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, work in preparation.
- [52] See F. Acampora, G. Amelino-Camelia, F. Buccella, O. Pisanti, L. Rosa and T. Tuzi, Nuovo Cim. A **108**, 375 (1995). G. Amelino-Camelia, F. Buccella and L. Rosa, arXiv:hep-ph/9405334. M. Abud, F. Buccella, D. Falcone, G. Ricciardi and F. Tramontano, Mod. Phys. Lett. A **15** (2000) 15 [arXiv:hep-ph/9911238].
- [53] G. F. Giudice and A. Romanino, arXiv:hep-ph/0406088. N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, arXiv:hep-ph/0409232. A. Arvanitaki, C. Davis, P. W. Graham and J. G. Wacker, arXiv:hep-ph/0406034. A. Pierce, Phys. Rev. D **70** (2004) 075006. S. h. Zhu, arXiv:hep-ph/0407072. C. Kokorelis, arXiv:hep-th/0406258. B. Mukhopadhyaya and S. SenGupta, arXiv:hep-th/0407225. R. Mahbubani, arXiv:hep-ph/0408096. M. Binger, arXiv:hep-ph/0408240. J. L. Hewett, B. Lillie, M. Masip and T. G. Rizzo, JHEP **0409** (2004) 070. L. Anchordoqui, H. Goldberg and C. Nunez, arXiv:hep-ph/0408284. K. Cheung and W. Y. Keung, arXiv:hep-ph/0408335. W. Kilian, T. Plehn, P. Richardson and E. Schmidt, arXiv:hep-ph/0408088. I. Antoniadis and S. Dimopoulos, arXiv:hep-th/0411032. M. A. Diaz and P. F. Perez, arXiv:hep-ph/0412066. P. F. Perez, arXiv:hep-ph/0412347.
- [54] B. Bajc and G. Senjanovic, arXiv:hep-ph/0411193.
- [55] It is interesting that the only other alternative is the fermionic triplet (actually triplets, at least two are needed similar to right-handed neutrinos)[9].
- [56] In supersymmetry this is not automatic, but depends on the Higgs superfields needed to break $SO(10)$ at M_{GUT} .
- [57] Type I can apparently be saved with CP phases, see [34]. For earlier work on type I see[35]